Mr. Michael T. Davis Calculus	Chapter 4 – Practice Test March 2, 2017

Name: Solutions

1. Find $\frac{dy}{dx}$ given $y = \sin^4(3x)$.

$$\frac{dy}{dx} = 4\sin^3(3x)\cdot 3\cos(3x) = 12\sin^3(3x)\cos(3x)$$

2. Determine the value of $\frac{dy}{dx}$ given $y = \sin^{-1}(3x)$

$$\frac{dy}{dx} = \frac{3}{\sqrt{1-9x^2}}$$

3. Determine the value of y' given $y = e^{5x}$

$$y' = 5e^{5x}$$

4. Find the slope of the curve of $xy^2 + x = 1$ at the point $\left(\frac{1}{2}, 1\right)$

$$y^2 + x\cdot 2yy' + 1 = 0$$

$$1 + \tfrac{1}{2}\cdot 2\cdot 1y' + 1 = 0$$

$$1 + y' + 1 = 0$$

$$y' = -2$$

slope $m = -2$

$$y^2 + x\cdot 2yy' + 1 = 0$$

$$y' = \frac{-1-y^2}{2xy}$$

$$y' = \frac{-1-1}{2\cdot\frac{1}{2}\cdot 1} = -2$$

5. Find the slope of the curve of $2y = x^2 + \sin y$ at the point $\left(\sqrt{2\pi}, \pi\right)$

$$2y' = 2x + \cos y\cdot y'$$

$$2y' - \cos y\, y' = 2x$$

$$y' = \frac{2x}{2-\cos y}$$

$$y' = \frac{2\sqrt{2\pi}}{2-\cos\pi} = \frac{2\sqrt{2\pi}}{2+1} = \frac{2\sqrt{2\pi}}{3}$$

6. Determine the value of y' given $y = 9^{-x}$

$y' = 9^{-x} \ln 9(-1) = -9^{-x} \cdot \ln 9$

7. Determine an equation for the line normal to the ellipse with equation $x^2 - xy + y^2 = 7$ at the point $(-1,2)$.

$2x - y - xy' + 2y \cdot y' = 0$

$2yy' - xy' = y - 2x$

$y' = \dfrac{y - 2x}{2y - x}$

at $(-1,2)$ $y' = \dfrac{2 - 2(-1)}{2(2) - (-1)}$

$y' = \dfrac{4}{5}$

$y - 2 = \frac{-5}{4}(x+1)$

8. Determine $\dfrac{dy}{dx}$ given the parametric curve defined by $x = 3\sin t$ and $y = 2\cos t$

$\dfrac{dx}{dt} = 3\cos t \qquad \dfrac{dy}{dt} = -2\sin t$

$\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{-2\sin t}{3\cos t} = -\dfrac{2}{3}\tan t$

9. Determine the value of $g'(x)$ given $g(x) = (\ln x)^2$

$g'(x) = 2(\ln x)\dfrac{1}{x} = \dfrac{2\ln x}{x}$

10. If $f(x) = \ln(x + 4 + e^{-3x})$, then determine the value of $f'(0)$

$f'(x) = \dfrac{1 - 3e^{-3x}}{x + 4 + e^{-3x}} \qquad f'(0) = \dfrac{1 - 3e^{0}}{0 + 4 + e^{0}} = \dfrac{-2}{5}$

11. Determine an expression for $\dfrac{dy}{dx}$ given $y = \tan^{-1}(\sqrt{x})$

$y' = \dfrac{\frac{1}{2}x^{-\frac{1}{2}}}{1 + x} = \dfrac{1}{2\sqrt{x}(1+x)}$

12. Determine the value of y' given $y=\log_5\left(\frac{1}{x}\right)$

$$y' = \frac{-1x^{-2}}{\frac{1}{x}\cdot\ln 5} = \frac{-x}{x^2\ln 5} = \frac{-1}{x\ln 5}$$

13. Determine an expression for $f'(x)$ given $f(x)=\tan^2\left(\sqrt{x}\right)$

$$f'(x) = 2\tan(\sqrt{x})\sec^2(\sqrt{x})\cdot\frac{1}{2}x^{-\frac{1}{2}}$$
$$= \frac{\tan(\sqrt{x})\sec^2(\sqrt{x})}{\sqrt{x}}$$

14. Determine an expression for y' given $y=x^3(2x-5)^4$

$$y' = 3x^2(2x-5)^4 + x^3\cdot 4\cdot 2(2x-5)^3$$
$$= 3x^2(2x-5)^4 + 8x^3(2x-5)^3$$

15. Determine the value of $\frac{dy}{dx}$ given $y=xe^x-x\ln x$

$$\frac{dy}{dx} = e^x + xe^x - \ln x - x\cdot\frac{1}{x}$$
$$= e^x + xe^x - \ln x - 1$$

16. Determine an expression for $\frac{d^2y}{dx^2}$ given $y=\cot(2-x)$

$$y' = -\csc^2(2-x)(-1) = +\csc^2(2-x)$$
$$y'' = 2\csc(2-x)(-\csc(2-x)\cot(2-x)(-1))$$
$$y'' = 2\csc^2(2-x)\cot(2-x)$$

17. Determine an expression for $\frac{d^2y}{dx^2}$ given $2x^3-3y^2=8$. Be sure to give $\frac{d^2y}{dx^2}$ in terms of x & y and not in terms of x, y & $\frac{dy}{dx}$.

$\frac{dy}{dx}=?$ $\quad \frac{6x^2}{6}-\frac{6y\cdot y'}{6}=\frac{0}{6}$ $\quad y'=\frac{-x^2}{-y}$ $\quad \frac{dy}{dx}=\frac{x^2}{y}$

$$\frac{d^2y}{dx^2}=\frac{y\cdot 2x-x^2y'}{y^2}=\frac{2xy-x^2\left(\frac{x^2}{y}\right)}{y^2}=\frac{\frac{2xy^2}{y}-\frac{x^4}{y}}{y^2}=\frac{2xy^2-x^4}{y^3}$$

18. Given $e=ye^x+x\ln y$, determine the slope of the curve at the point $(0,e)$

$0=y'e^x+ye^x+\ln y+x\cdot\frac{1}{y}y'$ $\quad$ at $(0,e)$

$-ye^x-\ln y=y'e^x+\frac{x}{y}y'$ $\quad y'=\frac{-e\cdot e^0-\ln e}{e^0+\frac{0}{e}}$

$\frac{-ye^x-\ln y}{e^x+\frac{x}{y}}=y'$ $\quad y'=\frac{-e-1}{1}=-e-1$

19. If $y=\cos^2\left(x^3+x^2\right)$, then $y'=$

$y'=2\cos(x^3+x^2)(-\sin(x^3+x^2))(3x^2+2x)$

a. $-2\sin\left(x^3+x^2\right)$

b. $-2\cos\left(x^3+x^2\right)\sin\left(x^3+x^2\right)$

c. $-2\cos\left(3x^2+2x\right)\sin\left(3x^2+2x\right)$

d. $-2\left(x^3+x^2\right)\cos\left(x^3+x^2\right)\sin\left(x^3+x^2\right)$

e. $-2\left(3x^2+2x\right)\cos\left(x^3+x^2\right)\sin\left(x^3+x^2\right)$

$y^2+x\cdot 2y\cdot y'-3x^2y-x^3y'=0$ $\quad$ (a) $y'=\frac{3x^2y-y^2}{2xy-x^3}$

$2xyy'-x^3y'=3x^2y-y^2$

Optional Extra Credit

(b) If $x=1$ then $y^2-y=6$ $\quad y^2-y-6=0$ $\quad (y-3)(y+2)=0$ $\quad y=3$ & $y=-2$

A curve in the xy-plane is defined by $xy^2-x^3y=6$.

a. Determine $\frac{dy}{dx}$.

at $(1,3)$ $\quad y'=\frac{3(1)^2(3)-3^2}{2(1)(3)-3^3}=0$ $\quad$ Tang. line Equation $y=0$

b. Find an equation for the tangent line at each point on the curve with x-coordinate $x=1$.

c. Find the x-coordinate of each point on the curve for which the tangent line is vertical.

at $(1,-2)$ $\quad y'=\frac{3(1)^2(-2)-(-2)^2}{2(1)(-2)-(1)^3}=\frac{-6-4}{-4-1}=\frac{-10}{-5}=2$

Tang. line Equation $\quad y+2=2(x-1)$